Analysis Two Dimension Heat Conduction in Functionally Graded Materials Using Finite Element Methods

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Abstract

Along with the progress of the industrial world, both the aviation industry, the health industry, the chemical industry, the electronics industry, and so on, the need for composite materials is increasing to meet market demand. Functionally Graded Materials (FGMs) are an advanced material class of composite materials that have material properties that vary from one point to another. In this study, two-dimensional heat conduction analysis will be conducted in FGM using the Finite Element Method (FEM). Three models gradation FGMs properties examined in the study, namely polynomial, Trigonometry, and Exponential. The response temperature of FGMs using gradation three models compared and analyzed. The optimum temperature distribution of four models built with the ANSYS software. The result is that heat conduction in trigonometric variations is very good, resulting in low-temperature values when compared to both of them. Then, the performance and efficiency obtained using FEM to analyze two-dimensional heat conductivity in FGMs is also very good.

Keywords: *FGMs, Heat Conduction, Finite Element Methods (FEM), ANSYS.*

A. INTRODUCTION

FGMs are usually associated with particulate composite materials, where the volume fraction of a particle varies one or more directions. One of the advantages of monotonous variation from the phase fraction of the constituent phase is the removal of the stress discontinuities. FGMs can also be developed using fiber-reinforced with a constant fraction of fiber volume, taking into account the production of optimal set properties or responses [1, 2].

In its development, several types of FGMs studied. In micromechanical-based elastic, two-phase models FGMs local interactions betweens particles while the effective material properties change gradually along with the gradation directions where the spherical or almost spherical particles are embedded in the isotropic matrix [3]. Besides, FGMs particulate fractional constituent phases and revised volumes in one vertical direction. In FGM, the temperature and placement function that corresponds to the fulfillment of boundary conditions at the edges is used to reduce the differential equation determined by thermomechanics for a set of ordinary differential equation pairs in the coordinate relationship. Thus, FGM can also have micro-approval [4]. FGM allows us to have a different architecture, using orthotropic motivation. Thermal barrier coating (TB) can reduce the surface temperature of metal from metal components. Thus, orthotropic FGMs have a microstructure and a flat column structure obtained from each plasma spray and electron document from the physical vapor deposition process [5].

There is a lot of heat transfer research in FGM in the industrial sector. In this sector, temporary heat transfer problems on the FGM strip are resolving with asymptotic solutions, theoretical treatments, and with the Local Boundary Integral (LBIM) method [6, 7, 8].

Transient heat transfer problems have also been resolving using the Boundary Element (BEM) approach compared to the Finite Element Method (FEM) analytical and simulation solutions [9]. The Boundary Element Method (BEM) is also used by [10, 11, 12]. Galerkin Boundary Element Method (GBEM) [13]. In another study, the combined approach of the BEM and Precise Integration Method (PIM) shows that PIBEM (a combination of PIM with the Radial Integration Limit Element Method "RIBEM") can achieve stable and accurate results for large time differences. But will produce small-time differences only in a different case. Thus, RIBEM-FD (RIBEM-Finite Difference) produces accurate results [14].

In other studies, sensitivity analysis of heat conductivity has been carried out on FGM using FEM [15, 16, 17, 18, 19, 20]. There are also other studies using the FEM method compared to direct and adjoin methods [21]. A multilevel FGM thermoelastic analysis study subjected to transient thermal shock. This study developed an asymmetric semi-analytic FE model using the theory of three-dimensional linear elasticity [22]. A year later, a study heat transfer of two dimensions by transient conduction in a hollow FG cylinder. In this study, heat transfer by conduction uses a limited length. The result for modeling and simulation of equations that govern the multilevel FEM used has several conventional FEM advantages [23]. And another one-dimensional FGM cylinder study was completed by [24] using Laplace domain analytic methods.

Other methods related to the solution of conduction heat conduction problems by using the BEM meshless method [25], Free Galerkin Element method (EFG) [26], meshless B-splines method [27,28], and that recently used the Numeric Manifold method (NMM) [29]. In addition to using numerical methods, transient conduction heat transfer problems can also use analytical methods [30-33].

A numerical procedure to presented to determine an optimal material distribution of Functionally Graded Materials (FGMs) for heat conduction problems. A volume fraction used the design of variable and material properties, which are assumed to be functions of temperature.

B. METHODS

A heat transfer analysis problem is essential for the fields of engineering and science that commonly founded in various technological applications such as electronic cooling, thermal insulation or heat conduction, and so on.

Heat conduction is the transfer heat energy from the object to another object or from one object to another without the transfer of particles or substances. Heat conduction can occur in gases, liquids, and solids.

Considering a two-dimensional bounded domain Ω with constant material parameters, the governing equation for transient heat conduction problems in isotropic media can be expressed as [13]:

$$\rho(\boldsymbol{x})c(\boldsymbol{x})\frac{\partial T(x,t)}{\partial t} = k(\boldsymbol{x})\nabla^2 T(\boldsymbol{x},t) + Q(\boldsymbol{x},t) \ \ in\ \Omega. \quad (1)$$

The Dirichlet boundary condition:

$$T = \bar{T}\ in\ \Gamma_1, \quad (2)$$

The Neumann boundary condition:

$$k_x\frac{\partial T}{\partial x}n_x + k_y\frac{\partial T}{\partial y}n_y = \bar{q}\ in\ \Gamma_2, \quad (3)$$

The Robin boundary condition:

$$k_x\frac{\partial T}{\partial x}n_x + k_y\frac{\partial T}{\partial y}n_y = h\left(T_f - T\right)\ in\ \Gamma_3, \quad (4)$$

where T represents temperature, $Q(\boldsymbol{x})$ is the heat-generation rate which is usually an explicit function of $\boldsymbol{x}$, k_x and k_y re specified thermal conductivities where x and y are the principal directions of the conductivity tensor, $\bar{T}$ and $\bar{q}$ are the prescribed temperature and the given heatflux on the corresponding boundaries, respectively, n_x and n_y are the direction cosines of the outward normal to the boundary surface, h is the convection heat transfer coefficient, T_f is the environmental temperature, and Γ_1, Γ_2 and Γ_3 are the boundaries to which the Dirichlet, Neumann and Robin conditions are applied. For simplicity, in this paper we only consider Dirichlet and Neumann boundary conditions.

The initial condition is:

$$T(\boldsymbol{x},t)|_{t=0} = T_0\ in\ \Omega. \quad (5)$$

The finite element method is a numerical procedure used to derive a solution to most engineering problems involving stress analysis, heat transfer, electromagnetic and fluid flow. There are many complex forms of domain problems that easy to solve.

In general, finite element methods (for some elements), are defined as:

$$[\boldsymbol{K}]\boldsymbol{T} = \boldsymbol{f}, \quad (6)$$

with $[\boldsymbol{K}]$ is a matrix condition, or it can also be spelled out as:

$$[\boldsymbol{K}] = K_{ij} = \int_\Omega \left[K_x\frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + K_y\frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}d\Omega\right] \quad (6a)$$

Where N_i dan N_j is a shape function of the Moving Least Squares (MLS) rows i dan column j, respectively, $\boldsymbol{T}$ is a vector representing nodal displacement and $\boldsymbol{f}$ is a vector that describes the nodal force and external force, or it can also be described as:

$$\boldsymbol{f} = F_i = \int_\Omega Q(x,y)N_i d\Omega + \int_\Gamma \bar{q}N_i d\Gamma. \quad (6b)$$

Using equation (2), the heat transfer equation using the FE method can be expressed as:

$$\boldsymbol{M}\dot{\boldsymbol{T}} + \boldsymbol{K}\boldsymbol{T} = \boldsymbol{f}, \quad (7)$$

with:

$$\boldsymbol{M} = \int_\Omega \rho c\boldsymbol{N}^T\boldsymbol{N}d\Omega, \quad (7a)$$

$$\boldsymbol{K} = \int_\Omega \boldsymbol{B_\theta}^{\boldsymbol{T}}\boldsymbol{B_\theta}K d\Omega, \quad (7b)$$

$$\boldsymbol{f} = -\int_\Gamma \boldsymbol{N^T}\bar{q}d\Gamma + \int_\Omega f\boldsymbol{N}^T d\Omega, \quad (7c)$$

$\boldsymbol{M}$ is the capacity matrix, $\dot{\boldsymbol{T}}$ is a vector that describes the nodal displacement over time, $\boldsymbol{K}$ is the matrix condition, $\boldsymbol{T}$ is a vector representing nodal displacement, $\boldsymbol{N^T}$ is the transpose matrix of the shape function and $\boldsymbol{f}$ is a vector that describes nodal forces and external forces.

C. RESULT AND DISCUSSION

To validate the method, some transient heat conduction problems two-dimensional are analyzed. The first example corresponds to a square domain composed of three different FGMs, corresponding to, respectively, a quadratic, an exponential, and a trigonometric variation of the material properties along the x-axis. To solve this example with the FEM approach, given by Eq. [6] is used. The second examples consist of a circular. A heat conduction problem in a circular cylinder made of functionally gradient material is analyzed. It is assumed that $T_i = 0^0C$ at the inner boundary and $T_o = 10^0C$ at the outer boundary. The thermal conductivity K is given by $K(x) = Ar + Br^2$ where r is the distance from the center of a circular cylinder, and inner and outer radii are $R_i = 5$ and $R_o = 10$ mm, respectively. the interpolated values in the case $A = 1$ and $B = 1$. And the third example. Geometry and boundary conditions, as shown in Fig. 1. The numerical response examples compared to the two-dimensional analytical solutions. And a temperature change all example calculated using software ANSYS 17.0 run on Asus A451L with OS Windows 8, processor Intel CORE-i5 NVIDIA GEFORCE 740M and RAM of 4GB. And the error value obtained, following the equation (28):

$$E_n = \sqrt{\frac{\sum_{i=1}^{NC}[u(x_i)-\hat{u}(x_i)]^2}{\sum_{i=1}^{NC}u(x_i)^2}}, \tag{8}$$

where $u(\boldsymbol{x}_i)$ and $\hat{u}(\boldsymbol{x}_i)$ represent the analytical and numerical solutions, respectively. It pointed out that for the heat conduction problems pose no analytical solution, numerical results use to other numerical methods available in the literature is used as a benchmark.

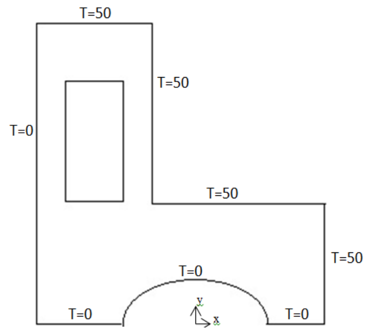


Figure 1. Geometry and Boundary Conditions

Table 1. Material Properties of the FGMs Square Problem

Variation	α	κ	$\bar{K}$	c_1	c_2	β	$K(\boldsymbol{x})$
Quadratic	0	5	5	1	2	0	$5(1+2x)^2$
Exponential	-5	5	5	1	0	1	$5\,exp(2x)$
Trigonometric	0.2	5	5	1	2	0.2	$5(\cos(0.2x)+2\sin(0.2x))^2$

NUMERICAL RESULTS

Square FGMs

Polynomial Variations

Figures 2a, 2b, and 2c show a temperature change calculated using software for the polynomial FGMs square shape with the nodal numbers 279, 1037, and 2275, respectively. An analytical solution as follows:

$$T(x,y,t) = \frac{T_1 x}{\sqrt{K(x)}L} + \frac{2T_1}{\sqrt{K(x)}}\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n\pi}\, x\, sin\left(\frac{n\pi x}{L}\right) exp\left(-\frac{n^2\pi^2}{L^2}\kappa t\right), \quad (9)$$

where $T_1 = \sqrt{\bar{K}}(1+\beta L)\bar{T}$. (9a)

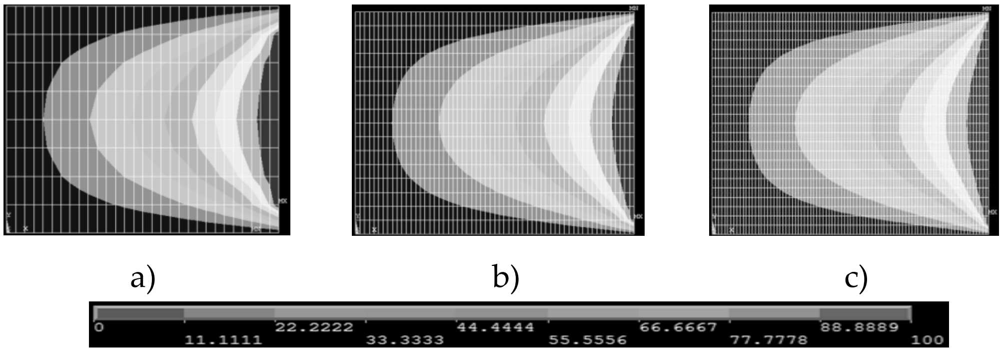


Figure 2. Plot contours from the temperature of the polynomial FGMs square shape with a) 279, b) 1037, and c) 2275 nodes

Eksponensial Variations

Figures 3a, 3b, and 3c show a temperature change calculated using software for the exponential FGMs square shape with the nodal numbers 279, 1037, and 2275, respectively. An analytical solution as follows:

$$T(x,y,t) = \bar{T}\frac{1-\exp(-2\beta x)}{1-\exp(-2\beta L)} + \sum_{n=1}^{\infty}\frac{2\bar{T}\exp(\beta L)n\pi\cos(n\pi)}{\beta^2L^2+n^2\pi^2}\, x\, sin\left(\frac{n\pi x}{L}\right) exp(-\beta x)\exp(-(\frac{n^2\pi^2}{L^2} + \beta^2)\kappa t). \quad (10)$$

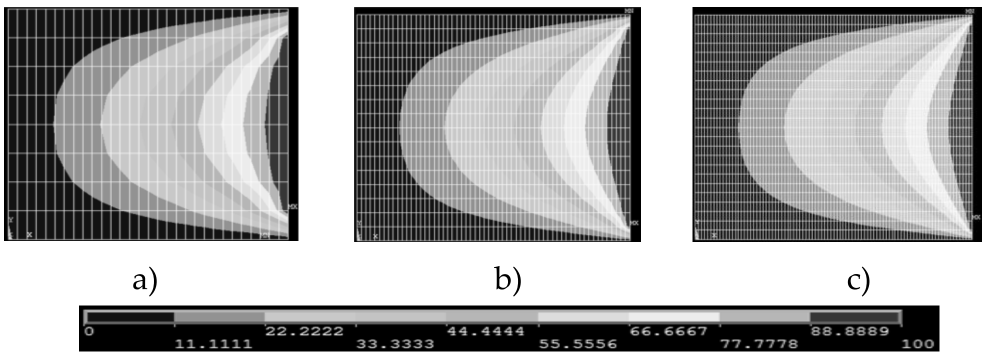


Figure 3. Plot contours from the temperature of the exponential FGMs square shape with a) 279, b) 1037, and c) 2275 nodes

Trigonometri Variations

Figures 4a, 4b, and 4c show a temperature change calculated using software for the trigonometry FGMs square shape at with the nodal numbers 279, 1037, and 2275, respectively. An analytical solution as follows:

$$T(x_1,x_2,t)=\frac{T_2\sin(\beta x_1)}{\sqrt{K(x_1)}\sin(\beta L)}+\frac{2T_2}{\sqrt{K(x_1)}}\sum_{n=1}^{\infty}\frac{\mathrm{n\pi cos}(n\pi)}{n^2\pi^2-\beta^2L^2}\;x\;sin\left(\frac{n\pi x_1}{L}\right)\exp(-(\frac{n^2\pi^2}{L^2}+\beta^2)\kappa t),\qquad(11)$$

Where $T_2=\sqrt{\bar{K}}(\cos(\beta L)+2\sin(\beta L))\bar{T}$. (11a)

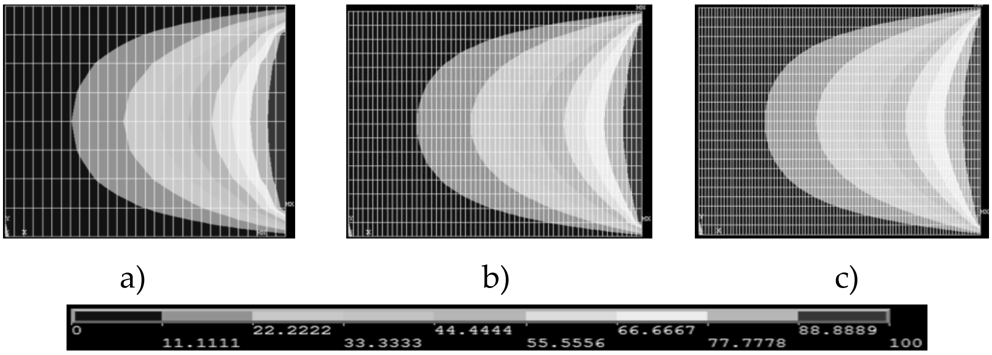


Figure 4. Plot contours from the temperature of the trigonometry FGMs square shape with a) 279, b) 1037, and c) 2275 nodes

Hollows Cylinder FGMs

Polinomial Variations

Figure 5 shows a temperature change calculated using the FE method compared to using an exact solution on the polynomial FGMs. The results show that there is almost the same result between the two. Because the FEM method is a manual solution developed based on exact solutions obtain. Fig.7 shows the temperature graph of time at different coordinate points (x,y). An analytical solution as follows:

$$T(r)=\frac{(T_i-T_o)\left[Bln\left\{\frac{(A+Br)R_i}{(A+BR_i)r}\right\}+\frac{A(r-R_i)}{R_ir}\right]}{Bln\left\{\frac{(A+BR_i)R_O}{(A+BR_O)R_i}\right\}+\frac{A(R_i-R_O)}{R_iR_O}}+T_i.\qquad(12)$$

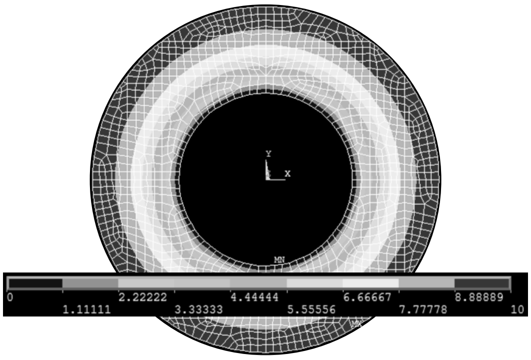


Figure 5. Plots of contours from the temperature of the polynomial FGMs with 761 nodes

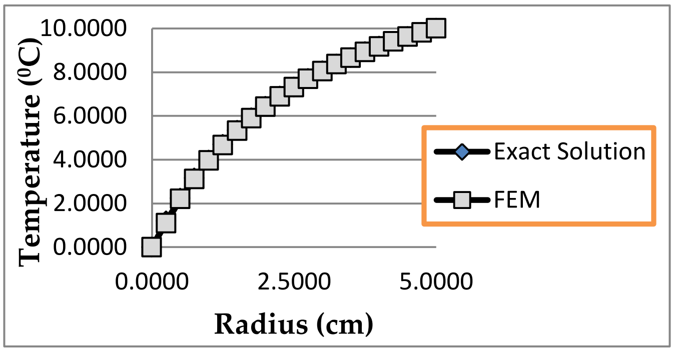


Figure 6. Comparison of FEM temperature with the exact solution

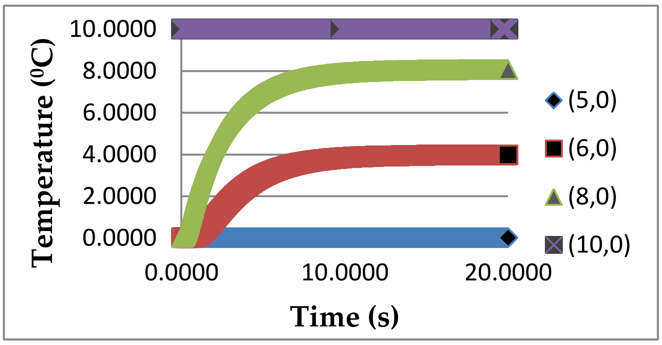
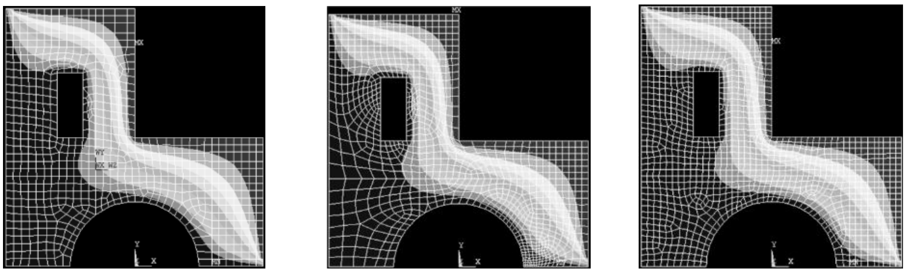


Figure 7. Graph of temperature versus time at some point coordinates (x, y) is different

Geometry Complicated FGMs

Polinomial Variations

Figures 8a, 8b, and 8c show a contour plot of polynomial FGMs on complicated geometries. The results obtained there is no analytical solution. Therefore, the results obtained only based on numerical solutions with nodal amounts of 828, 1065, and 1329, respectively.



a) b) c)

Figure 8. Plot contours from the temperature of the polynomial FGMs with a) 828, b) 1065, and c) 1329 nodes

Exponential Variations

Figures 9a, 9b, and 9c show a contour plot of exponential FGMs on complicated geometries. The results obtained there is no analytical solution. Therefore, the results obtained only based on numerical solutions with nodal amounts of 828, 1065, and 1329, respectively.

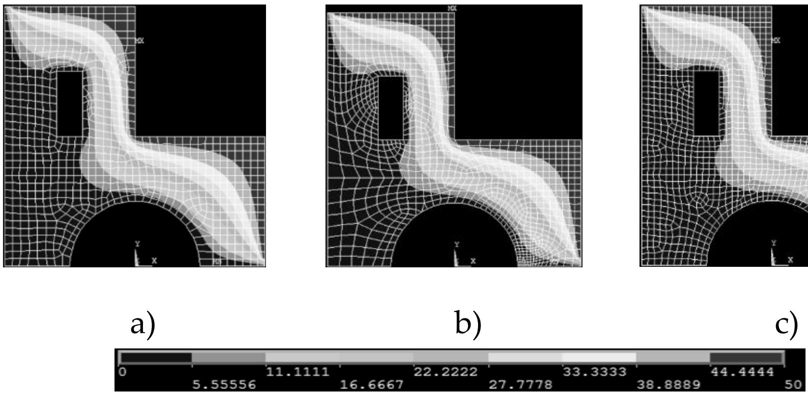


Figure 9. Plot contours from the temperature of the exponential FGMs with a) 828, b) 1065, and c) 1329 nodes

Trigonometry Variations

Figures 10a, 10b, and 10c show a contour plot of exponential FGMs on complicated geometries. The results obtained there is no analytical solution. Therefore, the results obtained only based on numerical solutions with nodal amounts of 828, 1065, and 1329, respectively.

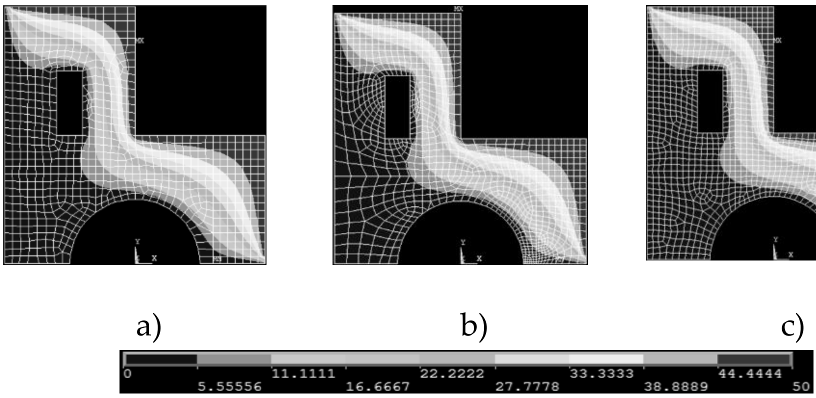


Figure 10. Plot contours from the temperature of the trigonometry FGMs with a) 828, b) 1065, and c) 1329 nodes

D. CONCLUSION

After conducting experiments on two-dimensional geometries such as cylindrical holes, squares, and complex geometries, with variations in the heat conduction properties of FGMs given quadratic, exponential, and trigonometry a conclusion is drawn. The result that FEM is the method recommended to analyze heat conduction in two-dimensional FGMs. The result is that heat conduction in trigonometric variations is very good, resulting in low-temperature values compared to both of them. Then, the performance and efficiency obtained using FEM to analyze two-dimensional heat conductivity in FGMs is also very good. The hollow cylinder geometry, the average temperature obtained $\mathrm{T} = 30.33447^{0}C$. Then in the square geometry $\mathrm{T} = 46.0835^{0}C$. And the last in complicated geometry is $\mathrm{T} = 25.2129^{0}C$. Then, the performance and efficiency obtained using FEM to analyze two-dimensional heat conductivity in FGMs is also very good, respectively. A geometric hollow cylinder with quadratic, exponential, and trigonometry variation with a total capital of 1379, the processing time was 434.6 s, 435 s, and 444 s, respectively. Then in the square geometry, we get the same average processing time, which is 37 s. And finally, in complicated geometry, we get the same average processing time, which is 35 s. In the geometry of a hollow cylinder with quadratic variations with a nodal number of 761, an average error value of 0.0019 obtained. Then in the hollow geometry cylinder, the quadratic variation with a nodal number 883, the average error value is 0.0013. And the last one in the hollow geometry cylinder, quadratic variation with the number of nodal 1379, the average error value is 0.0012.

REFERENCES

1. Birman, V. (1995). Stability of functionally graded hybrid composite plates. *Composites Engineering*, *5*(7), 913-921.
2. Birman, V. (1997). Stability of functionally graded shape memory alloy sandwich panels. *Smart Materials and Structures*, *6*(3), 278-286.
3. Yin, H. M., Sun, L. Z., & Paulino, G. H. (2004). Micromechanics-based elastic model for functionally graded materials with particle interactions. *Acta Materialia*, *52*(12), 3535-3543.
4. Vel, S. S., & Batra, R. C. (2002). Exact solution for thermoelastic deformations of functionally graded thick rectangular plates. *AIAA journal*, *40*(7), 1421-1433.
5. Kaysser, W. A., & Ilschner, B. (1995). FGM research activities in Europe. *Mrs Bulletin*, *20*(1), 22-26.
6. Jin, Z. H. (2002). An asymptotic solution of temperature field in a strip a functionally graded material. *International communications in heat and mass transfer*, *29*(7), 887-895.
7. Ootao, Y., & Tanigawa, Y. (2004). Transient thermoelastic problem of functionally graded thick strip due to nonuniform heat supply. *Composite Structures*, *63*(2), 139-146.
8. Sladek, J., Sladek, V., & Zhang, C. (2003). Transient heat conduction analysis in functionally graded materials by the meshless local boundary integral equation method. *Computational Materials Science*, *28*(3-4), 494-504.
9. Sutradhar, A., & Paulino, G. H. (2004). The simple boundary element method for transient heat conduction in functionally graded materials. *Computer Methods in Applied Mechanics and Engineering*, *193*(42-44), 4511-4539.

10. Sutradhar, A., Paulino, G. H., & Gray, L. J. (2005). On hypersingular surface integrals in the symmetric Galerkin boundary element method: application to heat conduction in exponentially graded materials. *International Journal for Numerical Methods in Engineering*, *62*(1), 122-157.
11. Abreu, A. I., Canelas, A., & Mansur, W. J. (2013). A CQM-based BEM for transient heat conduction problems in homogeneous materials and FGMs. *Applied Mathematical Modelling*, *37*(3), 776-792.
12. Ochiai, Y. (2004). Two-dimensional steady heat conduction in functionally gradient materials by triple-reciprocity boundary element method. *Engineering analysis with boundary elements*, *28*(12), 1445-1453.
13. Chen, J., Liu, Z., & Zou, Z. (2002). Transient internal crack problem for a nonhomogeneous orthotropic strip (Mode I). *International Journal of Engineering Science*, *40*(15), 1761-1774.
14. Yao, W., Yu, B., Gao, X., & Gao, Q. (2014). A precise integration boundary element method for solving transient heat conduction problems. *International Journal of Heat and Mass Transfer*, *78*, 883-891.
15. Bruch Jr, J. C., & Zyvoloski, G. (1973). A finite element weighted residual solution to one-dimensional field problems. *International Journal for Numerical Methods in Engineering*, *6*(4), 577-585.
16. Bruch Jr, J. C., & Zyvoloski, G. (1974). Transient two-dimensional heat conduction problems solved by the finite element method. *International Journal for Numerical Methods in Engineering*, *8*(3), 481-494.
17. Wang, B. L., & Mai, Y. W. (2005). Transient one-dimensional heat conduction problems solved by finite element. *International Journal of Mechanical Sciences*, *47*(2), 303-317.
18. Wang, B. L., & Tian, Z. H. (2005). Application of finite element–finite difference method to the determination of transient temperature field in functionally graded materials. *Finite elements in analysis and design*, *41*(4), 335-349.
19. Haghighi, M. G., Eghtesad, M., & Malekzadeh, P. (2008). Coupled DQ–FE methods for two dimensional transient heat transfer analysis of functionally graded material. *Energy conversion and management*, *49*(5), 995-1001.
20. Malekzadeh, P., Golbahar Haghighi, M. R., & Heydarpour, Y. (2012). Heat transfer analysis of functionally graded hollow cylinders subjected to an axisymmetric moving boundary heat flux. *Numerical Heat Transfer, Part A: Applications*, *61*(8), 614-632.
21. Chen, B., & Tong, L. (2004). Sensitivity analysis of heat conduction for functionally graded materials. *Materials & design*, *25*(8), 663-672.
22. Santos, H., Soares, C. M. M., Soares, C. A. M., & Reddy, J. N. (2008). A semi-analytical finite element model for the analysis of cylindrical shells made of functionally graded materials under thermal shock. *Composite Structures*, *86*(1-3), 10-21.

23. Asgari, M., & Akhlaghi, M. (2009). Transient heat conduction in two-dimensional functionally graded hollow cylinder with finite length. *Heat and Mass Transfer*, *45*(11), 1383-1392.
24. Babaei, M. H., & Chen, Z. (2010). Transient hyperbolic heat conduction in a functionally graded hollow cylinder. *Journal of Thermophysics and Heat Transfer*, *24*(2), 325-330.
25. Gao, X. W. (2006). A meshless BEM for isotropic heat conduction problems with heat generation and spatially varying conductivity. *International journal for numerical methods in engineering*, *66*(9), 1411-1431.
26. Zhang, X., Zhang, P., & Zhang, L. (2013). An improved meshless method with almost interpolation property for isotropic heat conduction problems. *Engineering Analysis with Boundary Elements*, *37*(5), 850-859.
27. De Boor, C. (1972). On calculating with B-splines. *Journal of Approximation theory*, *6*(1), 50-62.
28. Hidayat, M. I. P., Wahjoedi, B. A., Parman, S., & Yusoff, P. S. M. (2014). Meshless local B-spline-FD method and its application for 2D heat conduction problems with spatially varying thermal conductivity. *Applied Mathematics and Computation*, *242*, 236-254.
29. Zhang, H. H., Han, S. Y., & Fan, L. F. (2017). Modeling 2D transient heat conduction problems by the numerical manifold method on Wachspress polygonal elements. *Applied Mathematical Modelling*, *48*, 607-620.
30. Hosseini, S. M., Akhlaghi, M., & Shakeri, M. (2007). Transient heat conduction in functionally graded thick hollow cylinders by analytical method. *Heat and Mass Transfer*, *43*(7), 669-675.
31. Wang, H. M. (2013). An effective approach for transient thermal analysis in a functionally graded hollow cylinder. *International Journal of Heat and Mass Transfer*, *67*, 499-505.
32. Daneshjou, K., Bakhtiari, M., Alibakhshi, R., & Fakoor, M. (2015). Transient thermal analysis in 2D orthotropic FG hollow cylinder with heat source. *International Journal of Heat and Mass Transfer*, *89*, 977-984.